Measurement of Radio Frequency Permittivity of Biological Tissues with an Open-Ended Coaxial Line: Part I

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Abstract—**An open-ended coaxial line is analyzed as a sensor for *in vivo* measurement of the complex permittivity of biological substances. The measurement system, based on two computerized network analyzers operating at frequencies from 10 MHz to 1 GHz, is described. Experimental results are presented in a companion paper.**

I. Introduction

NONDESTRUCTIVE BROAD-BAND methods for the measurement of the permittivity of materials, particularly of biological materials at radio frequencies (RF), are of continuing interest. Knowledge of the dielectric properties of biological tissue is necessary in determining the energy deposition in living subjects (e.g., experimental animals) and representative models exposed to RF fields [1], [2]. This information is essential in evaluating potential exposure hazards and in effective and controllable induction of local hyperthermia in the treatment of cancer. Some biophysical interaction mechanisms of RF fields with biological systems can be inferred from the tissue permittivity [1].

Recently an increased interest has been observed in measurements of these properties *in vivo* using a coaxial impedance probe. This interest stems from two sources. First, some questions have arisen concerning the difference in the measured permittivity of the same tissue *in vivo* and *in vitro*. Second, a method suitable for *in vivo* measurements is nondestructive and may be used in other applications where special preparation of the test sample is undesirable. Also, this approach is usually broad band. The method may also offer important advantages in routine testing of properties of other substances, e.g., moisture content of food products.

A review of broad-band measurement techniques of the permittivity of biological tissues is given in [3]. Two configurations have been identified as suited for *in vivo* measurements. These are a short monopole antenna [4], [5], and an open-ended coaxial line [3]. A short monopole with a limited-size ground plane inserted into the test tissue was previously used to measure the RF dielectric properties *in vivo* at frequencies between 10 MHz and 10 GHz [4], [5]. Use was made of an antenna modeling theorem relating the input impedance of an antenna immersed in a lossy dielectric medium to the antenna's input impedance in free space. A minicomputer-controlled network analyzer was employed to measure the input impedance.

The objective of the research reported here is to analyze an open-ended coaxial line sensor for *in vivo* measurement of the permittivity, and to develop a precision measurement system. A concept of the optimum capacitance [3], [6], [7], was used in selecting the sensor dimensions and in the analysis of measurement uncertainties. The measurement system was based on the HP 8407 and HP 8410 network analyzers. Reference materials of known dielectric properties and selected biological tissues were measured in the frequency range 0.01–1 GHz. The experimental results are analyzed in a companion paper [11].

II. Principle of Operation

The measurement system consists of a sensor and a computer-controlled network analyzer. The sensor translates changes in the permittivity of a test sample into changes in the input reflection coefficient of the sensor. These impedances are then measured by the network analyzer. An open coaxial line, placed in contact with a test sample (Fig. 1(a)), is used as a sensor. The sample must be homogeneous within a volume sufficiently large to simulate a slab which is electrically infinite in size. The equivalent circuit of the sensor consists of two elements (Fig. 1(b)), a lossy capacitor $C(\hat{\epsilon})$ ($C(\hat{\epsilon}) = C_0\hat{\epsilon}$, where C_0 is the capacitance when the line is in air), and a capacitor C_f which accounts for the fringing field in the Teflon. This equivalent circuit is valid at frequencies where the dimensions of the line are small compared with the wavelength so that the open end of the line is not radiating and all the energy is concentrated in the fringe or reactive near field of the line. At higher frequencies the value of the capacitance C_0 increases with frequency [8], due to the increase in the evanescent TM modes being excited at the junction discon-

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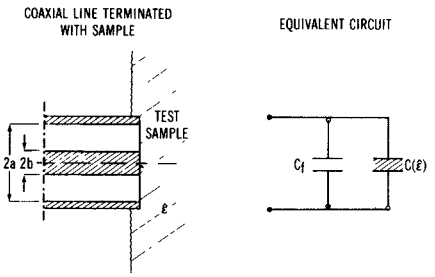

Fig. 1. Open-ended coaxial line sensor. (a) Geometry. (b) Equivalent circuit.

tinuity. When the evanescent modes are taken into account, an expression $C_0 + Af^2$, where A is a constant dependent on the line dimensions, should be used instead of C_0. Furthermore, when a high dielectric constant material is placed at the interface, these modes may become propagating modes (e.g., the TM_{01} mode propagates when $\lambda_\epsilon < 2.03\,(a-b)$, where λ_ϵ is the wavelength in the external dielectric and a, b are the line dimensions shown in Fig. 1(a). A parallel conductivity term may have to be added to the equivalent circuit of Fig. 1(b) to account for this radiation. In this study where the frequencies are sufficiently low that the equivalent circuit of Fig. 1(b) is applicable, the input reflection coefficient at the plane of the discontinuity can be calculated from

$$\hat{\Gamma} = \Gamma e^{j\phi} = \frac{1 - j\omega Z_0\left[C(\hat{\epsilon}) + C_f\right]}{1 + j\omega Z_0\left[C(\hat{\epsilon}) + C_f\right]}. \tag{1}$$

Solving for the permittivity we get

$$\hat{\epsilon} = \frac{1-\hat{\Gamma}}{j\omega Z_0 C_0(1+\hat{\Gamma})} - \frac{C_f}{C_0}. \tag{2}$$

It has been shown [3], [6], [7], that the greatest accuracy in determining the permittivity for a given accuracy of the reflection coefficient measurement is obtained when

$$C_0 = \frac{1}{\omega Z_0 \sqrt{\epsilon'^2 + \epsilon''^2}}. \tag{3}$$

This expression strictly holds only when the uncertainties in the magnitude and phase of the reflection coefficient are approximately the same, i.e., $\Delta\phi \simeq \Delta\Gamma/\Gamma$. In other cases this is still a good compromise as the optimum C_0 is different for ϵ' and ϵ'', these values being smaller and larger or vice versa than the value given by (3).

The total capacitance in air, $C_T = C_0 + C_f$, is a function of the line dimensions, the permittivity of the dielectric filling the line, and the frequency. The capacitance C_T can be calculated analytically (8) and also determined experimentally. However, one must resort to experimental methods to separate C_0 and C_f.

III. Probe Selection

The following criteria for the probe selection should be considered: 1) the optimum capacitance condition; 2) size, especially for *in vivo* measurements; 3) compatibility with liquid, semiliquid and solid samples; and 4) range of operating frequencies.

TABLE I
Optimum Capacitance (pF) for a Probe in Various Materials

MATERIAL	FREQUENCY (GHz)				
	0.01	0.05	0.1	0.5	1.0
Muscle	0.25	0.25	0.25	0.08	0.035
Fat	0.8	0.8	0.8	0.45	0.45
Water 25 °C	4	0.8	0.4	0.08	0.04
0.02 N NaCL 25 °C	0.37	0.37	0.37	0.08	0.04
0.08 N NaCL 25 °C	0.19	0.19	0.19	0.08	0.04
0.25 N NaCL 25 °C	0.08	0.07	0.07	0.055	0.035

TABLE II
Capacitance of Open Coaxial Lines

Line	a (cm)	b (cm)	C_T (pF)
14 mm, air	0.7145	0.3102	0.14
7 mm, air	0.35	0.1520	0.079
8.3 mm, teflon	0.362	0.1124	0.055
6.4 mm, teflon	0.2655	0.0824	0.041
3.6 mm, teflon	0.1499	0.0455	0.027
2.2 mm, teflon	0.0838	0.0255	0.016

Approximate values of the optimum capacitance, calculated from (3), for several biological substances and reference materials are given in Table I. The total capacitance C_T for a few commercially available coaxial lines, is listed in Table II.

From a comparison of the values presented in Tables I and II it is evident that smaller Teflon lines, when used as probes, offer nearly optimum measurement conditions for high water content tissues such as muscle, brain, liver, kidney, etc., at higher frequencies (0.5–2 GHz). However, at lower frequencies and for low moisture content tissues, e.g., fat, a 14-mm line or a greater size line would provide better measurement accuracy. Airlines pose some problems in this application, as a thin plastic window has to be incorporated to prevent a liquid or semiliquid sample from getting into the line. If such a window is used, the calibration procedure must account for its presence.

The sensor capacitance can be slightly increased by extending the center conductor of the coaxial line, forming a short monopole [4], [5]. There are some limitations to this approach, however. First, only a relatively small increase in the capacitance can be obtained. Second, if a short antenna does not have a ground plane, establishing an effective short circuit at its end (necessary for calibration purposes) is difficult. However, a ground plane is undesirable for *in*

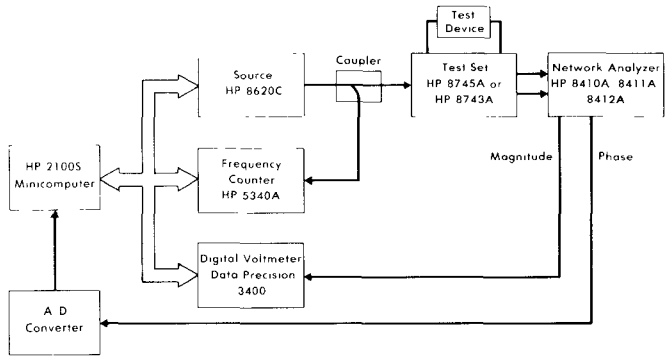

Fig. 2. Block diagram of semiautomated network analyzer system.

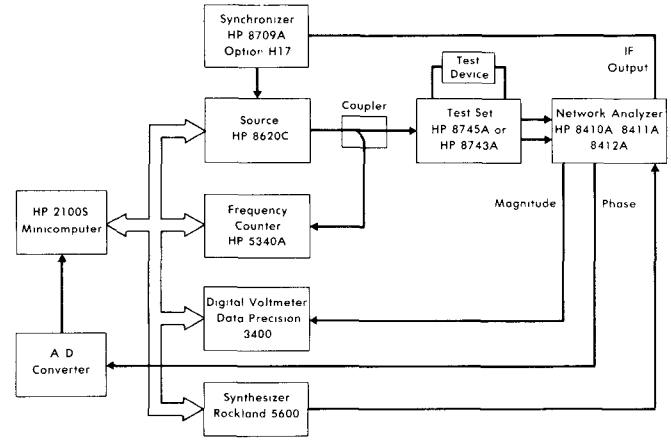

Fig. 3. Block diagram of semiautomated network analyzer system with phase-lock option.

vivo measurements as it impedes the probe placement in the test tissue. Finally, the test medium should conform closely to the shape of the antenna. Even a minute pocket of air in the region of the highest electric field can produce a large error.

Generally, small diameter lines are more compatible with *in vivo* measurements. They introduce a smaller perturbation in the living organism and provide information about the permittivity of a small tissue volume. As previously mentioned, Teflon-filled lines are better suited than air lines for measurements of liquid or semiliquid substances. Open coaxial lines are not well suited for the measurement of solid materials unless it is possible to have the sample prepared with a very flat and smooth surface to ensure good contact with the line.

Finally, the required range of frequencies of operation governs the probe selection. In this investigation aimed at measuring the permittivity of tissues with high water content at frequencies between 10 MHz and 1 GHz, a probe was selected with an outer diameter of 0.83 cm (0.325 in).

IV. Measurement System

Two automated network analyzer systems were used in this study. The first system was based on a Hewlett Packard (HP) 8410A network analyzer and covers the frequency range 0.1–12.4 GHz. The second system was based on an HP 8407 network analyzer and covered the range 0.001–0.11 GHz. In this study, measurements were made from 0.01 to 1.0 GHz using the two systems with duplicate measurements at 0.1 GHz.

The microwave network analyzer system is similar to the system described in Hewlett Packard Application Note No. 221 [9] with two exceptions. Our system was controlled by an HP 2100 minicomputer rather than a calculator, and phase information is taken from the HP 8412A phase-magnitude unit rather than an HP 8414A polar display unit. The block diagram of the system is shown in Fig. 2.

The use of the HP 8412A unit for the measurement of phase has the disadvantage of ambiguities and large errors near phase angles of $\pm 180°$. However, our system averages over 100–200 repetitive measurements and when the variance becomes large (as it will for a phase near $\pm 180°$), a request is printed for operator intervention to change the phase offset setting on the 8412 front panel. After resetting, the measurements can continue.

The other network analyzer system (not illustrated) is very similar to the microwave system, employing, in fact, the identical software except for the source controller. The HP 8601A RF source is not directly programmable, but it can be controlled through the use of an analogy signal. Using an iterative set/read procedure, the frequency can be set within 10 kHz.

An optional phase-locking subsystem [10], shown in Fig. 3, was employed in this study for the microwave network

analyzer system. At each frequency the HP 8620C source is programmed to the test frequency in the normal manner. Next, the synthesizer (Rockland 5600) is programmed to a specific submultiple of the local oscillator value required to down-convert the test frequency to the receiver's 20.278-MHz intermediate frequency (IF). The IF is sampled by an HP 8709A (option H-17) synchronizer and compared with its internal 20.278 oscillator. The error signal developed is applied to fine tune the source to the required test frequency.

With the phase-lock system, dramatic improvements in frequency accuracy and repeatability are achieved. An accuracy of one part in 10^7 is typical with repeatability within 100 Hz. The phase-lock system also completely eliminates errors due to "harmonic skip" [9], [10].

V. Uncertainty Analysis

General expressions giving the uncertainties in the dielectric constant and loss factor were previously derived as functions of uncertainties in the measured parameters (reflection magnitude and phase) and values of the permittivity [6]. For the purpose of the analysis presented in this work it is more convenient to express them as functions of the reflection coefficient rather than the permittivity. If the term C_f/C_0 is neglected, the following equations are obtained for the uncertainties:

$$\frac{\Delta\epsilon'}{\epsilon'}=\left\{\left(\frac{\Delta C_0}{C_0}\right)^2+\left(\frac{\Delta Z_0 2}{Z_0}\right)+\left(\frac{1-\Gamma^2}{1+2\Gamma\cos\phi+\Gamma^2}\times\frac{\Delta\Gamma}{\Gamma}\right)^2+\left(\frac{1-2\Gamma/\cos\phi+\Gamma^2}{1+2\Gamma\cos\phi+\Gamma^2}\times\frac{\Delta\phi}{\tan\phi}\right)^2\right\}^{1/2} \quad (4)$$

$$\frac{\Delta\epsilon''}{\epsilon''}=\left\{\left(\frac{\Delta C_0}{C_0}\right)^2+\left(\frac{\Delta Z_0}{Z_0}\right)^2+\left(\frac{4\Gamma+2\cos\phi+2\Gamma^2\cos\phi}{(1-\Gamma^2)(1+2\Gamma\cos\phi+\Gamma^2)}\Delta\Gamma\right)^2+\left(\frac{2\Gamma\sin\phi}{1+2\Gamma\cos\phi+\Gamma^2}\Delta\phi\right)^2\right\}^{1/2} \quad (5)$$

$$\frac{\Delta\tan\delta}{\tan\delta}=\left\{\left(\frac{\Delta\phi}{\tan\phi}\right)^2+\left(\frac{1+\Gamma^2}{1-\Gamma^2}\times\frac{\Delta\Gamma}{\Gamma}\right)^2\right\}^{1/2}. \quad (6)$$

The uncertainty in the frequency has been neglected since in the worst case ($\Delta f=10$ kHz at 10 MHz) it is much smaller than other uncertainties ($\Delta f/f<0.001$). It is interesting to note that the uncertainty in the loss tangent (tan δ) is not affected by the uncertainties in the capacitance and characteristic impedance.

The estimated uncertainty in a product of C_0 and Z_0 is 1 percent (since C_0 is determined experimentally for each probe, variations in Z_0 are accounted for because the permittivity depends on C_0Z_0). The estimated uncertainties resulting from systematic errors in the network analyzer systems are $\Delta\Gamma=0.003$, $\Delta\phi=0.3°$. These have been determined through comparison of measurements of a 3-dB attenuator/short assembly with this system and with a fully automated HP 8542 network analyzer system. In addition to the systematic errors there are nonsystematic errors due to the measurement systems and the probe. Nonsystematic errors are defined as those incapable of being characterized [10]. These include repeatability of discontinuities introduced by connectors after connecting and disconnecting, temperature drift, noise, nonperfect connector of the probe, and variations of the characteristic impedance along the probe line (see [11, section II]), dirt, imperfect contact with the sample, and inhomogeneities of the measured substance.

TABLE III
Estimated Uncertainties in the Dielectric Properties Measured with an 8.3-mm Probe, Using $\Delta\Gamma=0.003$ and $\Delta\phi=0.3°$ in [4] and [5]

Uncertainty	$\Delta\varepsilon'/\varepsilon'$			$\Delta\varepsilon''/\varepsilon''$		
Frequency (MHz)	10	100	1000	10	100	1000
Distilled Water	0.2	0.03	0.013	>1.0	>1.0	0.08
0.02 N NaCL Solution	0.2	0.025	0.013	0.03	0.03	0.03
0.08 N NaCL Solution	0.15	0.025	0.013	0.014	0.014	0.017
0.25 N NaCL Solution	0.07	0.02	0.012	0.012	0.012	0.013
Methanol	0.6	0.06	0.012	>1.0	0.75	0.02
Low Water Content tissue	0.1	0.025	0.013	0.03	0.03	0.03
High water Content tissue	0.05	0.025	0.013	0.014	0.014	0.016

The expected uncertainties in the dielectric parameters of typical reference materials and biological tissues measured by our system when systematic errors are considered are summarized in Table III. The uncertainty in the dielectric constant depends primarily on the accuracy of measurement of the phase angle, and, more specifically, for the given accuracy, on the value of that angle. The uncertainty decreases rapidly as the angle approaches 90°, which occurs when the probe capacitance approaches the optimum value. Similarly, the uncertainty in the loss factor is primarily affected by the uncertainty in the magnitude of the reflection coefficient for all values of the phase angle. The uncertainty in the capacitance-characteristic impedance value affects the overall measurement accuracy of both parameters when the capacitance value is close to the optimum value.

VI. Conclusions

An open-ended coaxial line, used for the measurement of the permittivity *in vivo*, over a limited frequency range can be represented by a simple equivalent circuit consisting of basically one capacitor. This convenient approximation is valid only if the relative dimensions of the line are such that changes of the capacitance with frequency and a parallel conductance can be neglected. This is the case for

a 0.83-cm (0.325-in) 50-Ω Teflon line employed as sensor at frequencies below 1 GHz.

Computerized network analyzer systems provide good measurement accuracy of the reflection coefficient when a standard error correction technique is used.

Selection of proper line dimensions to satisfy the optimum capacitance condition for a given dielectric material is critically important if good accuracy of measurements is expected. In measurements of biological tissues, a compromise between the line diameter and shape and the optimum capacitance has to be achieved.

The probe selected was capable of providing accurate measurements of the permittivity of biological tissues at frequencies from 50 MHz to 1 GHz, with gradual deterioration at lower frequencies. These probes could also be used at higher frequencies since the optimum capacitance condition would still be approximately satisfied. However, the equivalent circuit employed would no longer be valid. At higher frequencies the use of a smaller size line would appear to be more desirable since it would satisfy the simple equivalent circuit model.

Experimental results and a comparison of the estimated uncertainties with those expected due to the systematic errors as well as discussion of the nonsystematic errors are given in the companion paper [11].

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